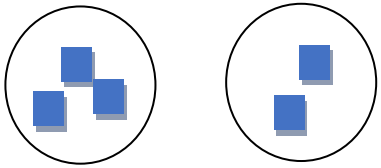
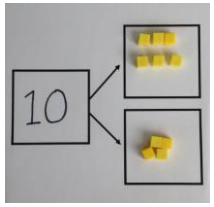
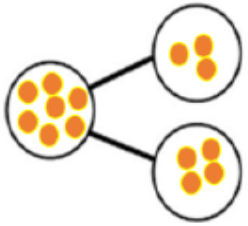
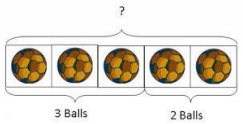
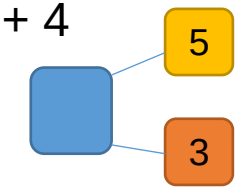
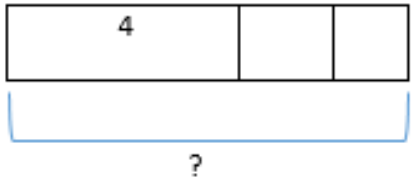
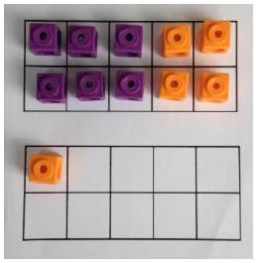
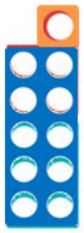
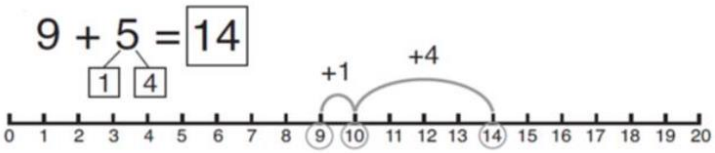
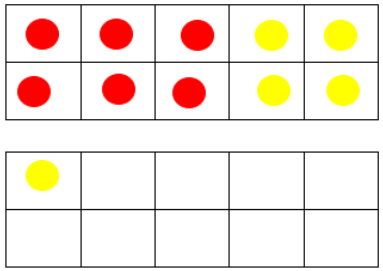
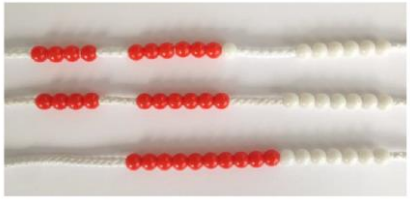
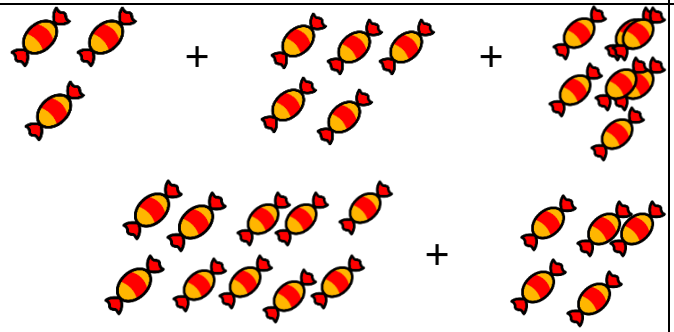
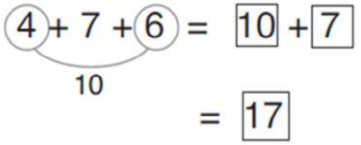
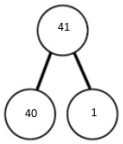
Progression in Calculations

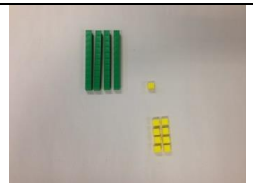
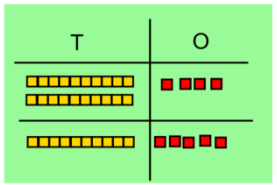
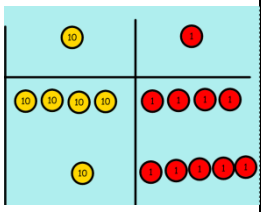
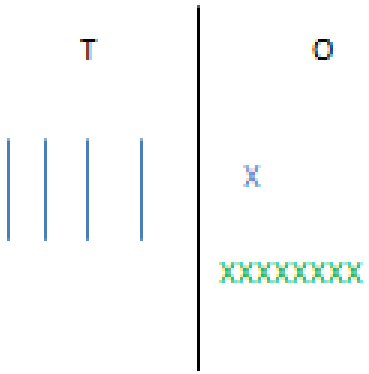
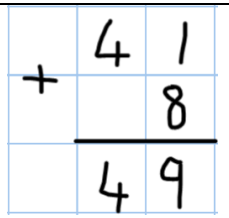
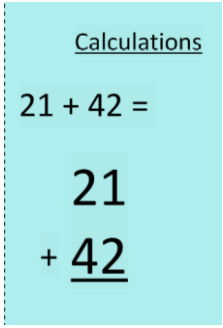
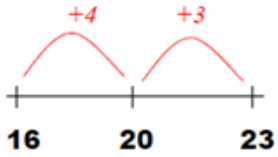
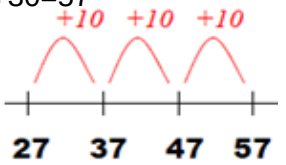
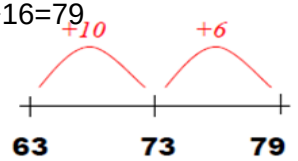
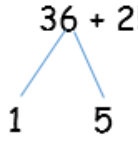
Addition

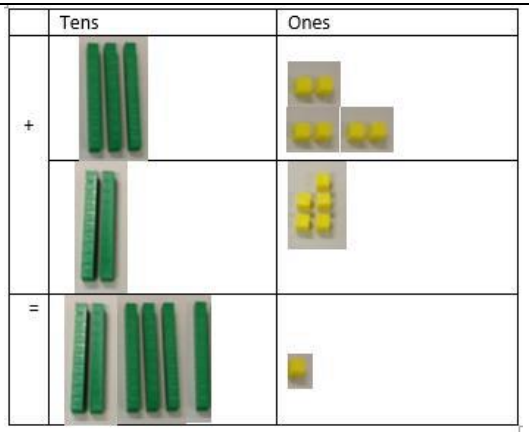
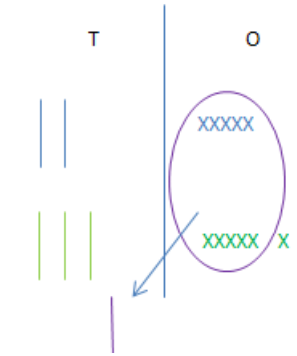
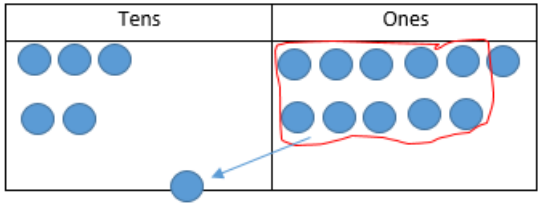
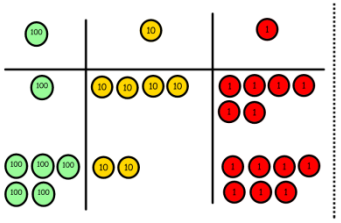
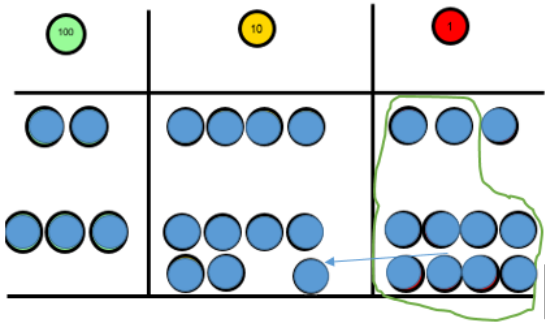
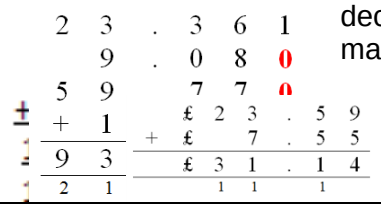
Key vocabulary – sum, total, parts and wholes, plus, add, altogether, more than, ‘is equal to’, ‘is the same as’

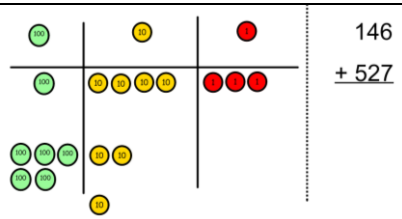
(Year group) Objective and Strategies	Concrete	Pictorial	Abstract
One more	Using a range of resources including (unifix blocks, counters, bead strings)	 Finding 1 more	Using number lines or counting on in head to find 1 more
Combining two groups	  Using a range of resources	 Using pictures to show two groups and counting them together	E.g. 4+1= 5  Using pictures. EXC: beginning to use number sentences to represent addition.

Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar.  Use a variety of resources	  Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Counting on using number lines by using cubes, numicon or bead strings.  	A bar model which encourages the children to count on. 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2  This can progress all the way to counting on using 2 digit numbers and greater. (year2)
Regrouping to make 10 by using ten frames and counters/cubes or using numicon.	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.	Use pictures or a number line. Regroup or partition the smaller number to make 10.	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

	 	   Children to draw the ten frame and counters/cubes	Children to develop an understanding of equality e.g. 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 +5 = ☐ + 4
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
2-digit + 1/2 digit, column method- no regrouping and counting	**TO + O using base 10**. Continue to develop understanding of partitioning and place value 41 + 8	Children to represent the concrete using a particular symbol, e.g. lines for 10s and dots/crosses for ones.	41 + 8  1 + 8 = 9 40 + 9 = 49

on. (See counting on for concrete and pictorial)	 **Progress onto TO + TO** using base 10 24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	 See also bar model image.	  16+7 = 23  27+30=57  63+16=79 
Column method- regrouping	**TO + TO using base 10.** Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25	This could be done one of two ways.	Looking for ways to make 10  30+20=50 5+5=10 50+10+1=61

		 	
Column method- regrouping	Use Base 10 or place value counters. Make both numbers on a place value grid.  146 + 527 Add up the ones and exchange 10 ones for one 10.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  If the children are completing a word problem, draw a bar model to represent what it's asking them to do.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 As the children move on, introduce 536 + 85 621 1 1  decimals



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

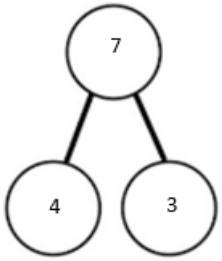
This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

?	
243	368

with the same number of decimal places and different. Money can be used here.

Fluency variation, different ways to ask children to solve 21 + 34:



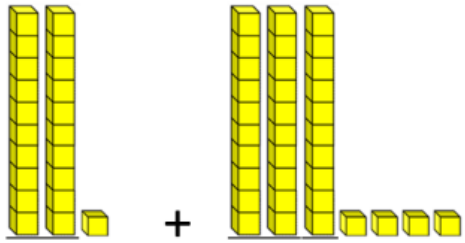
?	
21	34

Sam saved £21 one week and £34 another. How much did he save in total?

21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)

21
+34
___

21 + 34 =
□ = 21 + 34
What's the sum of twenty one and thirty four?

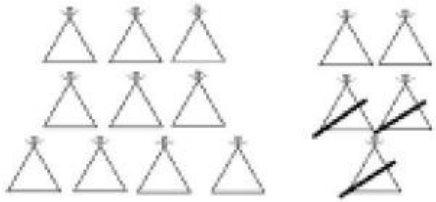
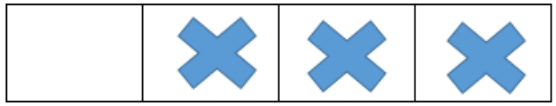
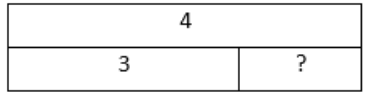
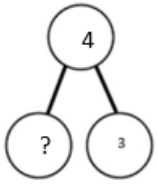


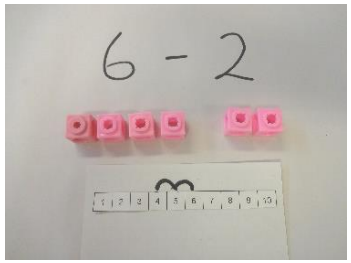
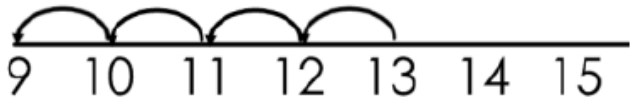
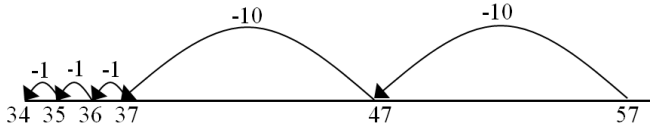
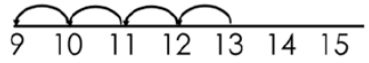
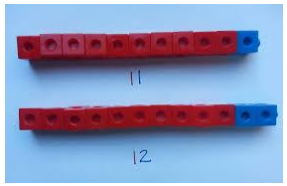
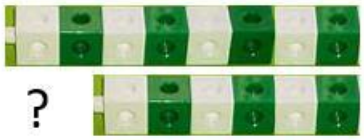
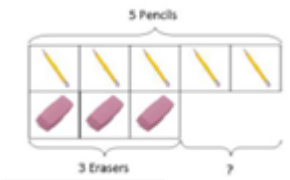
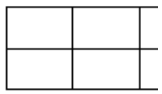
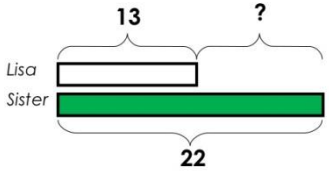
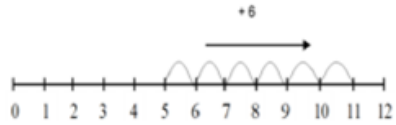
Always use missing digit problems too:

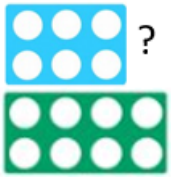
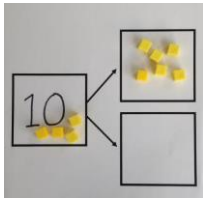
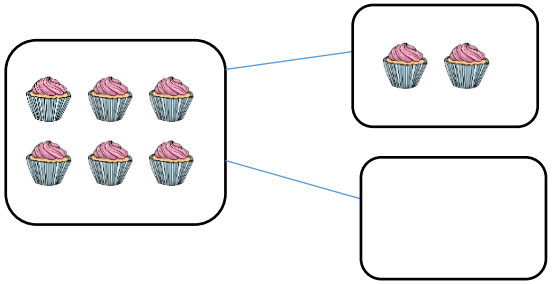
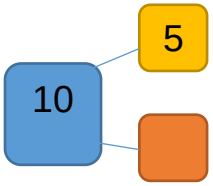
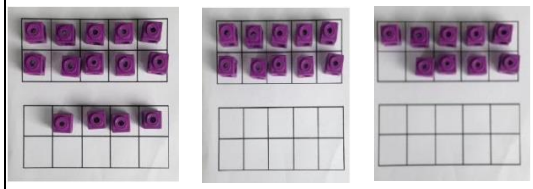
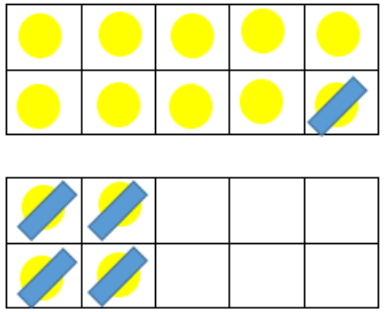
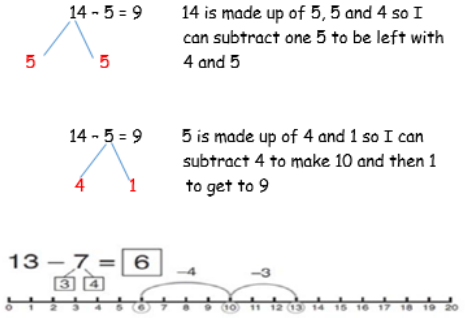
Tens	Ones
10 10	1
10 10 10	?
?	4

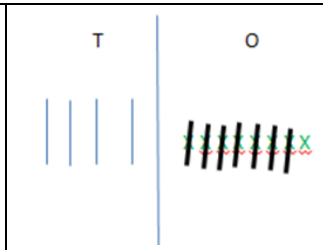
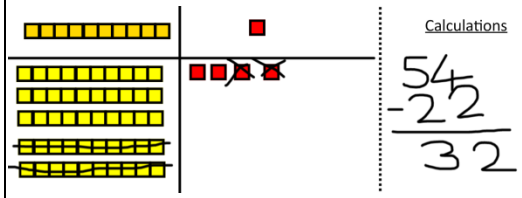
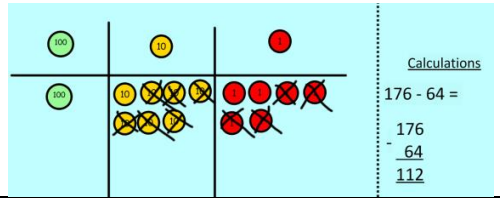
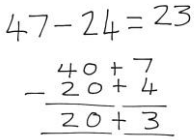
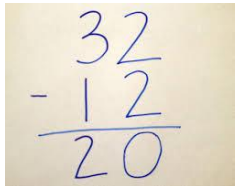
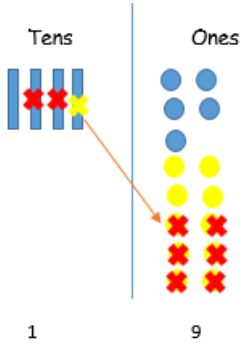
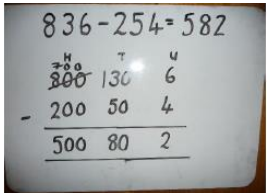
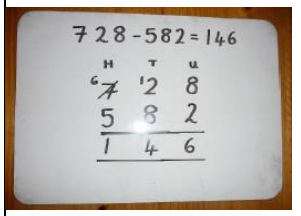
Subtraction

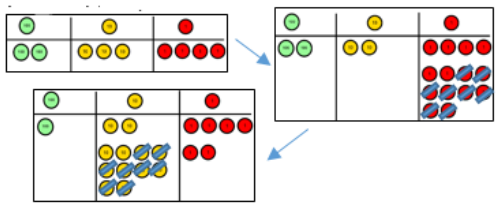
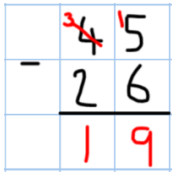
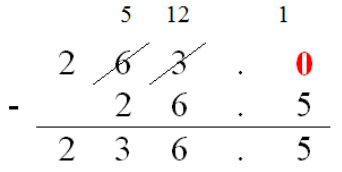
Key Vocabulary – take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'

(Year group) Objective and Strategies	Concrete	Pictorial	Abstract
1 less	Using a range of resources including (unifix blocks, counters, bead strings)	 Finding 1 less	Using number lines or counting on in head to find 1 less
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. Rather than crossing out, the children will physically remove the objects.   4-2=2	Cross out drawn objects to show what has been taken away.  Use of the bar model 	8 – 2 =  = 8 - 2  

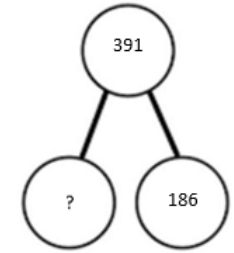
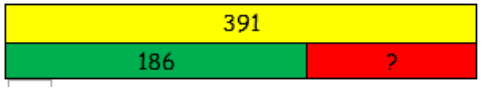
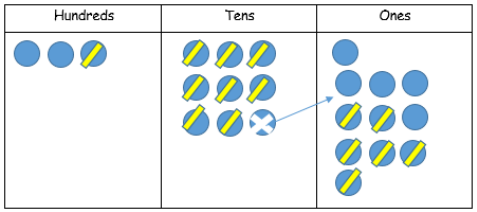
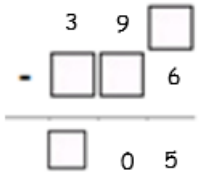
Counting back	Count back using number lines or number tracks  Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4 	 Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Count back on a number line or number track    This can progress all the way to counting back using two 2 digit numbers. (Year 2)
Find the difference	Compare amounts and objects to find the difference. Use cubes, numicon, and other objects.  Use cubes to build towers or make bars to find the difference  	 Use basic bar models with items to find the difference  **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  Draw bars to find the difference	 Count on to find the difference. Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. Find the difference between 8 and 6.

		between 2 numbers. Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX	8 – 6, the difference is ? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	Using numicon or 10 frames 14 – 5 =  	Children to present the 10 frame pictorially. 	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g. 

Column method without regrouping	Use base 10 (2-digit -1-digit, 2-digit – 2-digit and beyond) 	   Draw the Base 10 or place value counters alongside the written calculation to help to show working.	 This will lead to a clear written column subtraction 
Column method with regrouping	Using Base 10 and having to exchange. 45-26  1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens. Using place value counters 234-88	Represent the base 10 pictorially  Once the children have had practice with the concrete, they should be able to apply it to any subtraction. Like the other pictorial representations, children to represent the counters.	 Children can start their formal written method by partitioning the number into clear place value columns.  It's crucial that the children

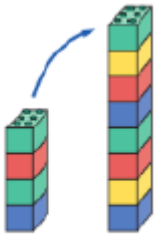
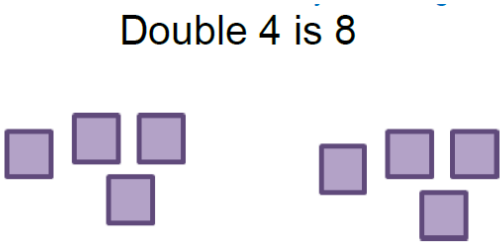
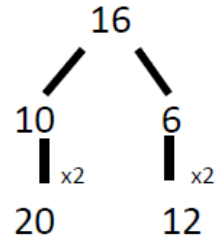
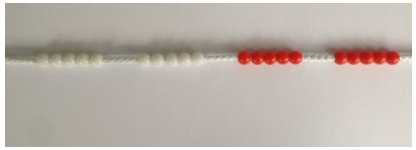
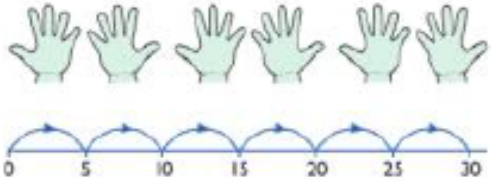
			understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15  This will lead to an understanding of subtracting any number including decimals. 

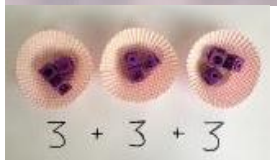
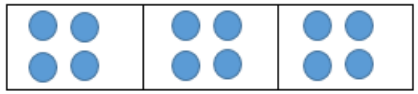
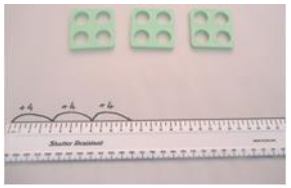
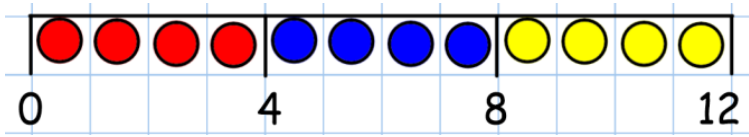
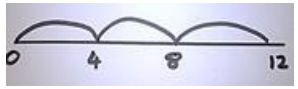
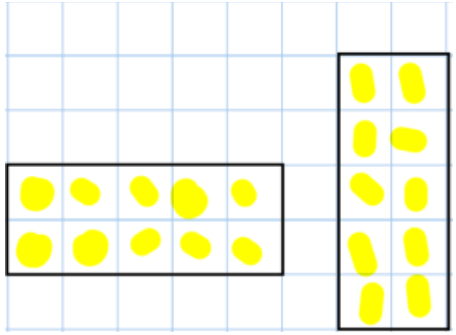
Fluency variation, different ways to ask children to solve 391-186

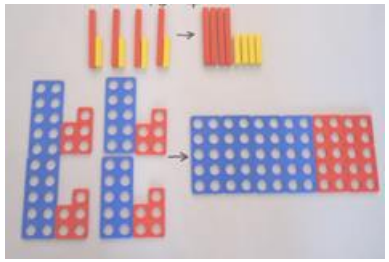
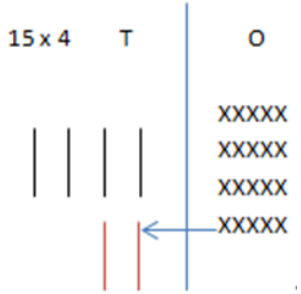
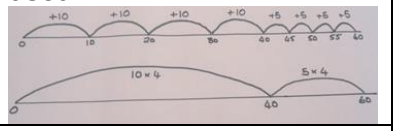
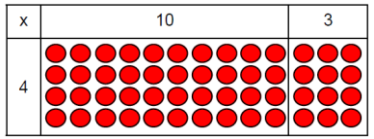
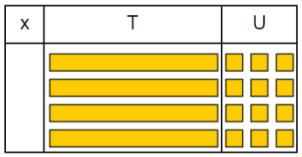
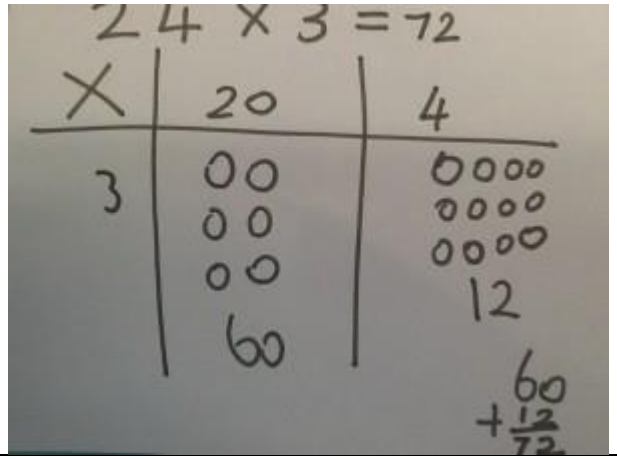
  	Raj spent £391, Timmy spent £186. How much more did Raj spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?	391 – 186 □ = 391 – 186 391 -186 Find the difference between 391 and 186 Subtract 186 from 391. What is 186 less than 391?	What's the calculation? What's the answer?  

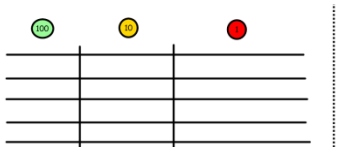
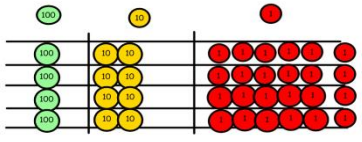
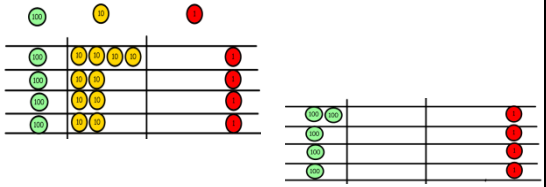
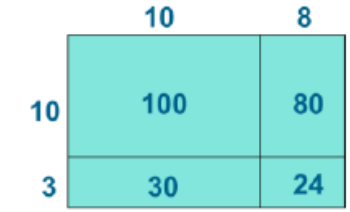
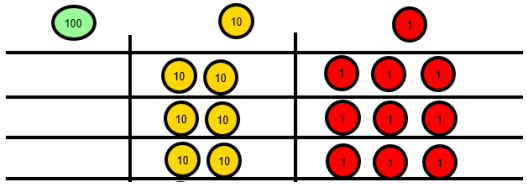
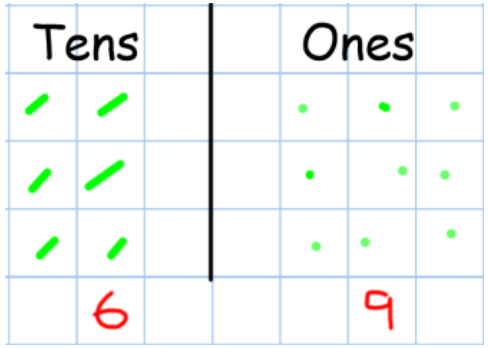
Multiplication

Key vocabulary- double, times, multiplied by, the product of, groups of, lots of, ‘is equal to’, ‘is the same as’

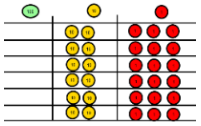
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated grouping/ addition	    Use different objects to add equal groups.	Children to represent the practical resources in a picture, e.g. XX XX XX XX XX XX **Use of a bar model for a more structures method.** 	3 x 4 4 + 4 + 4
Use number lines to show repeated groups	3 x 4  	Represent this pictorially alongside a number line, e.g. 	Abstract number line 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Children to draw the arrays. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

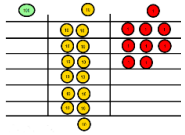
Partition to multiply	Use numicon or Base 10. 4 x 15 	Children to represent the concrete manipulatives in a picture, e..g Base 10 can be represented like: 	Children to be encouraged to show the steps they have taken 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Grid Method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | 7 | 210 | 35 | 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	 Calculations 4 x 126 Fill each row with 126.  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.  Then you have your answer.		 X | 1000 | 300 | 40 | 2 10 | 10000 | 3000 | 400 | 20 8 | 8000 | 2400 | 320 | 16
Column multiplication	Use place value counters or base 10 (at the first stage – no exchanging) 3 x 23 Make 23, 3 times. See howmany ones, then how many tens. 	Children to represent the counters in a pictorial way  	Children to record what it is they are doing to show understanding 3 x 23 20 3 3 x 20 = 60 3 x 3 = 9 60 + 9= 69 The aim is to get to the

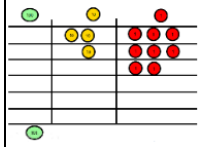
6 x 23



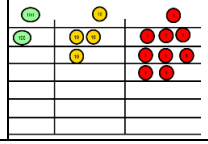
Step 1: get 6 lots of 23



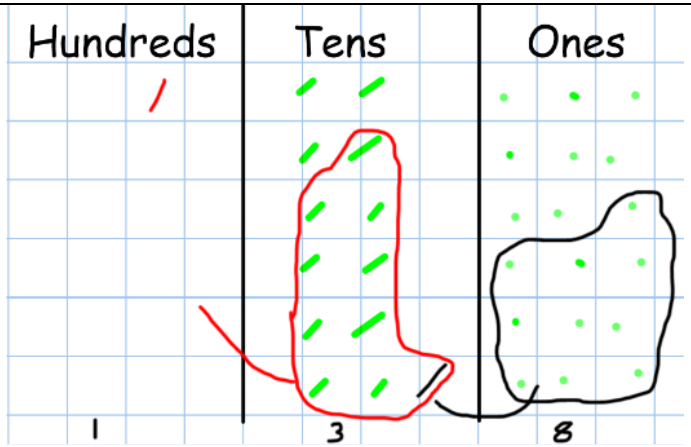
Step 2: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten….



Step 3: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred…



Step 4- what do I have I each column?



formal method but the children need to understand how it works.

6 x 23 =

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline {\scriptstyle 1\ 1} \end{array}$$

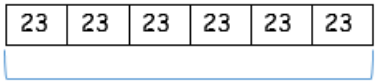
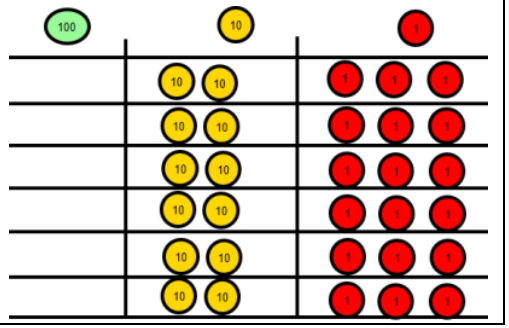
When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract:

To get 744 children have solved 6 x 124
To get 2480 they have solved 20 x 124

$$\begin{array}{rrrr} & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline & {\scriptstyle 1} & {\scriptstyle 2} & \\ & 7 & 4 & 4 \\ 2 & 4 & 8 & 0 \\ \hline 3 & 2 & 2 & 4 \\ \hline {\scriptstyle 1} & {\scriptstyle 1} & & \end{array}$$

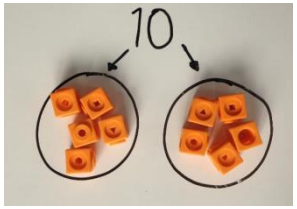
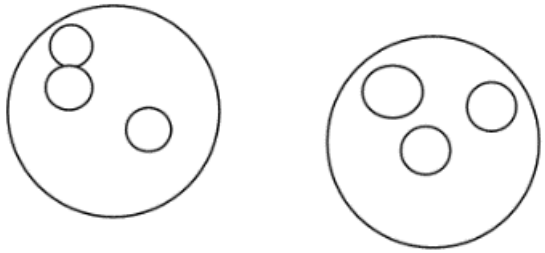
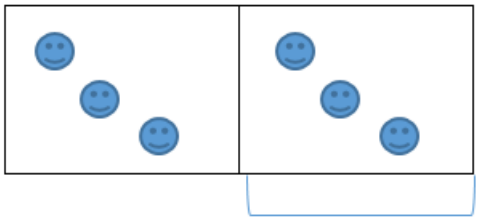
Answer: 3224

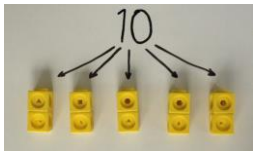
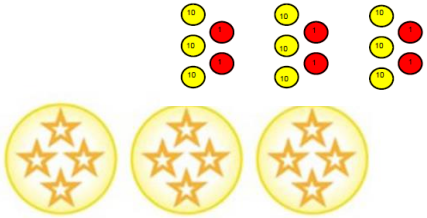
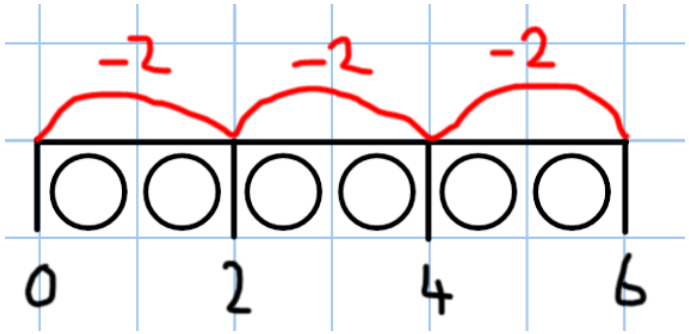
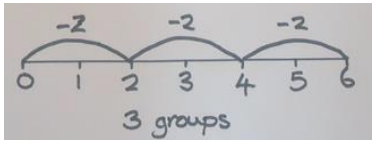
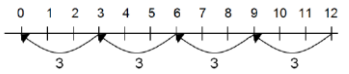
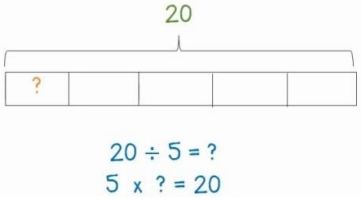
Fluency variation, different ways to ask children to solve 6 x 23

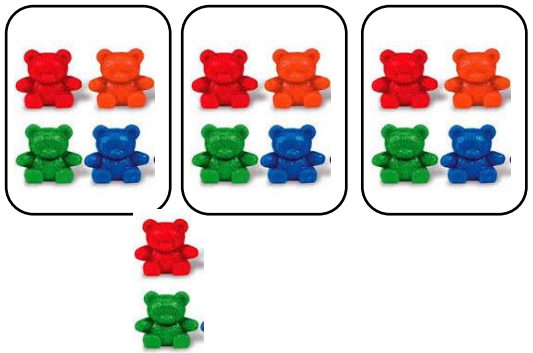
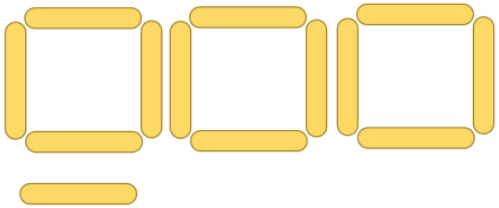
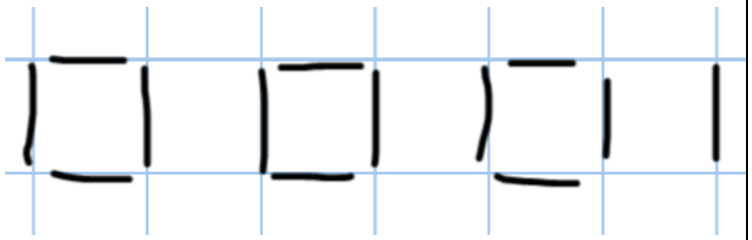
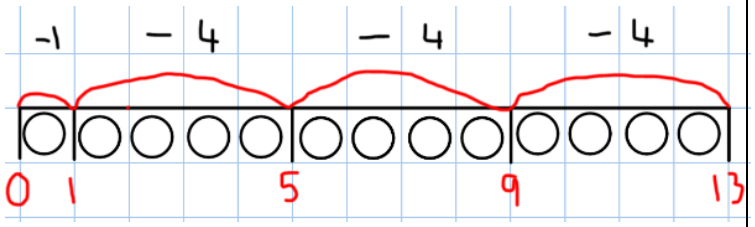
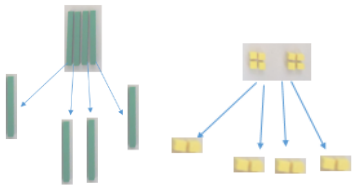
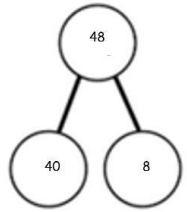
 With the counters, prove that 6 x 23 = 138 Why is 6 x 23 = 32 x 6?	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? Tom saved 23p three days a week. How much did he save in 2 weeks?	Find the product of 6 and 23 6 x 23 = □ = 6 x 23 6 × 23 23 × 6	What's the calculation? What's the answer? 

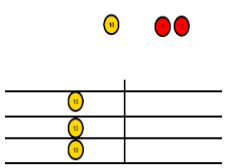
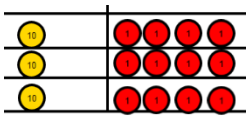
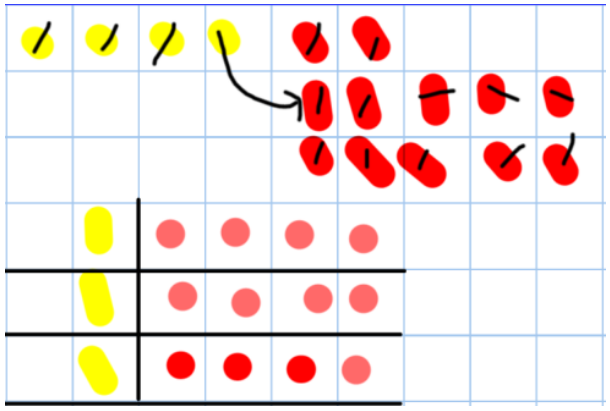
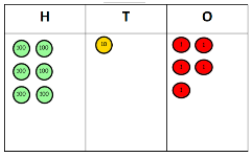
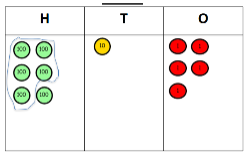
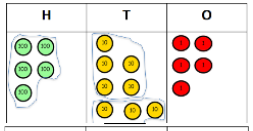
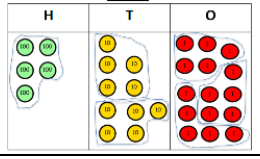
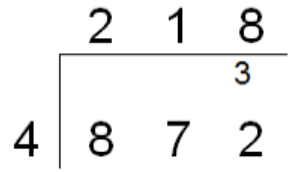
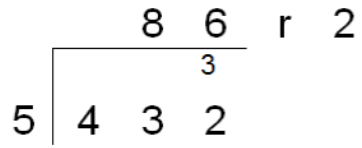
Division

Key vocabulary – share, group, divide, divided by, half, 'is equal to', 'is the same as'

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	 This can also be done in a bar so all 4 operations have a similar structure: 	6 ÷ 2 = 3 What's the calculation? 3 | 3

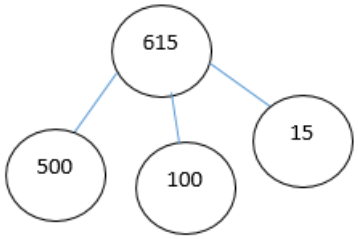
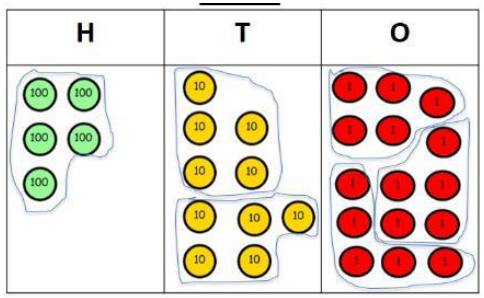
Division as grouping/ repeated subtraction	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Children to draw a picture of objects grouped.  6÷2=  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group? Use a number line to show jumps in groups. The number of jumps equals the number of groups.   Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 

Division with a remainder 2d ÷1d	14 ÷ 3 = Divide objects between groups and see how much is left over  Use of lolly sticks to form wholes  13 ÷4 =	Children to represent the resources they use in a pictorial way.    	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷4 = 3 r1 
Division using base 10 2d÷1d (no remainders) SHARING	48 ÷4 = 12  Start with the tens.	Children to represent the base 10 and sharing pictorially.	48 ÷4  4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12

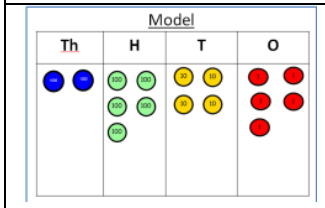
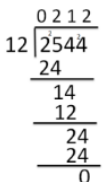
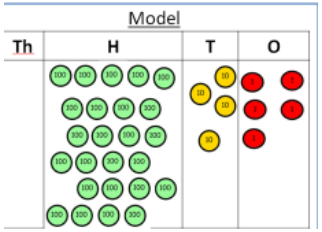
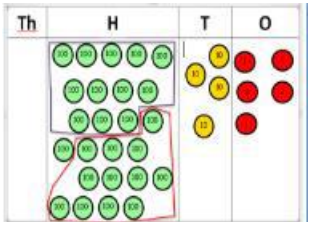
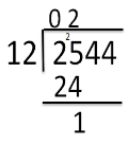
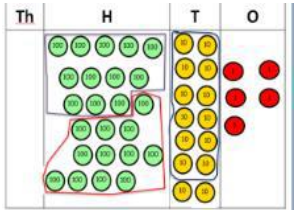
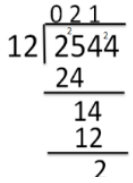
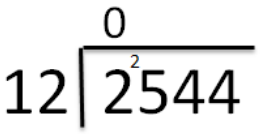
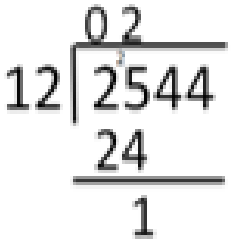
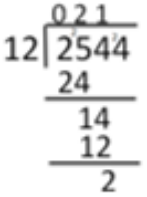
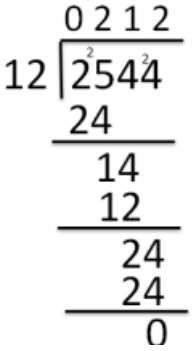
Sharing with place value counters	42 ÷3 = 14  1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?  Exchange the ten for 10 ones and share out 12 ones		42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14
'Bus stop' method	Use grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- ***this can also be done using sharing!*** 615 ÷ 5  Step 1: make 615  Step 2: Circle your groups of 5  Step 3: Exchange 1H for 10T and circle groups of 5  Step 4: exchange 1T for 10ones and	This can easily be represented pictorially, till the children no longer to do it. It can also be done to decimal places if you have a remainder!	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder. 

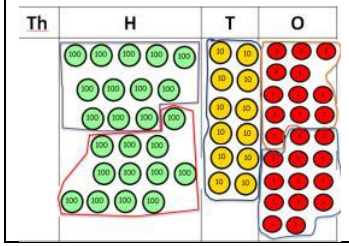
	circles groups of 5		Finally move into decimal places to divide the total accurately. $$\begin{array}{r} 1\,4\,.\,6 \\ 35\overline{)\,5\,1\,1\,.\,0} \\ \quad\;\; 16\;\;\;\;21 \end{array}$$

Fluency variation, different ways to ask children to ask to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?  	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5\overline{)615}$ 615 ÷ 5 = □ = 615 ÷ 5 How many 5's go into 615?	What's the calculation? What's the answer? 

Long division

Concrete	Pictorial	Abstract
  2544 ÷ 12 How many groups of 12 thousands do we have? None  Exchange 2 thousand for 20 hundreds.   How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  	Children to represent the counters, pictorially and record the subtractions beneath.	 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.  Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.  Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.  Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.  Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2		